

**FACTOR ANALYSIS OF GEOGRAPHIC COORDINATES
AT POINTS OF THE CHANNEL OF A SMALL RIVER ON
SPACE IMAGES**

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ABSTRACT

The purpose of the article is the analysis of asymmetric wavelets in binary relations between three coordinates at 290 characteristic points from the source to the mouth of the small river Irovka. The hypsometric characteristic is the most important property of the relief. The Irovka River belongs to a low level, at the mouth it is 89 m high, and at the source it is 148 m above sea level. Modeling of binary relations with latitude, longitude, and height has shown that local latitude receives the greatest quantum certainty. In this case, all paired regularities received a correlation coefficient of more than 0.95. Such a high adequacy of wave patterns shows that geomorphology can go over to the wave multiple fractal representation of the relief. The Irovka River is characterized by a small anthropogenic impact, therefore, the relief over a length of 69 km has the natural character of the oscillatory adaptation of a small river to the surface of the Vyatka Uval from its eastern side. This allows us to proceed to the analysis of the four tributaries of the small river Irovka, as well as to model the relief of the entire catchment basin of 917 km². The greatest adequacy with a correlation coefficient of 0.9976 was obtained by the influence of latitude on longitude, that is, the geographical location of the relief of the river channel with respect to the geomorphology of the Vyatka Uval. In second place with a correlation of 0.9967 was the influence of the height of the points of the channel of the small river on local longitude and it is also mainly determined by the relief of the Vyatka Uval. In third place was the effect of latitude on height with a correlation coefficient of 0.9859. And in last sixth place is the inverse effect of altitude on local latitude in the North-South direction.

Keywords: *river, channel, latitude, longitude, altitude*

INTRODUCTION

One of the possible ways of analyzing the Earth's remote sensing is to use the approaches of mathematical landscape morphology — the direction of landscape science, which studies the quantitative laws of building mosaics formed on the earth's surface and develops methods for their mathematical analysis [1].

The article [2] gives a brief overview of the experience of using harmonic analysis for morphometric characteristics of the relief. Up to 10 harmonics with constant period and amplitude of oscillation were obtained. The Fourier transform can be used for the classification and zoning of the earth's surface according to its

harmonic characteristics that determine the specifics of the topographic division of the site.

In recent years, interest in ecology has noticeably increased, and attention to small rivers has increased. Small rivers are the most vulnerable link in river systems, which is associated with their low water content and low erosion-transporting ability. Hence the special sensitivity of small riverbeds to anthropogenic impacts: the construction of earthen dams, bridges, local water intake and discharge of untreated (usually) wastewater, deforestation and plowing of watersheds. Any of these actions causes irreversible changes in the small river system until its death [3].

The fractal distribution of rivers by length in a river network has certain advantages over other morphometric indicators that are used to describe river networks. The fractal approach significantly increases the possibility of a quantitative description of river and erosion-channel networks [4].

Using satellite images, we measured coordinates (latitude, longitude, altitude) [5] according to methodological recommendations [6]. The characteristic points from the source to the mouth were selected on the line of the small river rod by changes in the longitudinal profile of at least 10-150 at the channel of the small river. Then, according to the measurement results, a coordinate table is compiled to identify patterns according to the method [7].

The goal is the analysis of asymmetric wavelets in binary relations between three coordinates at 290 characteristic points from the source to the mouth of the small river Irovka.

MATERIALS AND METHODS

Table 1 shows the results of measurements of three coordinates in the form of a fragment.

Table 1. Characteristic coordinates points of the channel of the river Irovka

The hypsometric characteristic is one of the most important properties of the relief. By elevation of the land surface above sea level, a low-lying (absolute height from 0 to 200 m) relief is distinguished [8].

The Irovka River has a height of 89 m at the mouth, and 148 m at the source (Fig. 1).

| Point rank | Latitude α , minute | Longitude β , minute | Height h , m |
|------------|----------------------------|----------------------------|----------------|
| 0 | 0 | 17.39 | 59 |
| 1 | 0.02 | 17.50 | 52 |
| 2 | 0.19 | 17.62 | 48 |
| ... | ... | ... | ... |
| 287 | 23.84 | 2.019 | 4 |
| 288 | 23.87 | 2.035 | 2 |
| 289 | 23.89 | 2.017 | 0 |

According to the hypsometric picture in Figure 1, Irovka flows inside a rectangle 23.89 minutes long (local latitude) and 18.89 minutes wide (local longitude). Sudden changes in the channel curvature in terms of 290 characteristic points gave several wave equations. Next, consider binary relations.

Oscillations (wavelet signals) are written by the wave formula [7] of the form

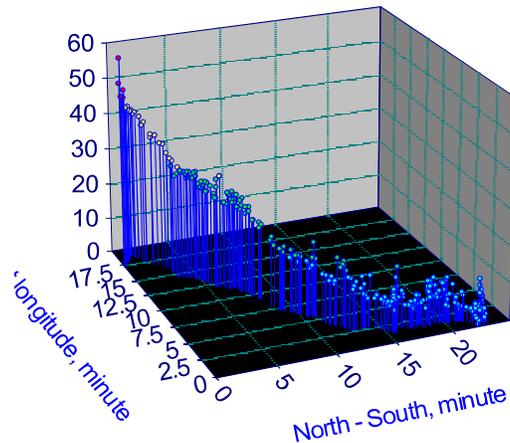


Fig. 1. Spatial hypsometry of the river

$$y_i = A_i \cos(\pi x / p_i - a_{3i}), \quad A_i = a_{1i} x^{a_{2i}} \exp(-a_{3i} x^{a_{4i}}), \quad p_i = a_{5i} + a_{6i} x^{a_{7i}}, \quad (1)$$

where y - is the indicator (dependent factor), i - is the number of the component of the model (1), m - is the number of terms in (1), x - is the explanatory variable (influencing factor), a_1 - a_8 - are the parameters (1) that take values during structural-parametric identification in CurveExpert-1.40, A_i - the amplitude (half) of the wavelet (axis y), p_i - the half-wave period (axis x).

According to the formula (1) with two **fundamental physical constants** ℓ (the number of Neper or the number of time) and \mathcal{T} (the number of Archimedes or the number of space), a **quantized wavelet signal** is formed from within the studied phenomenon and/or process. The concept of wavelet signal allows us to abstract from the physical meaning of many statistical series of measurements and consider their additive decomposition into components in the form of a sum of individual wavelets.

A signal is a material carrier of information. And we understand information as **a measure of interaction**. The signal can be generated, but its reception is not required. A signal can be any physical process or part of it. It turns out that the change in the set of unknown signals has long been known, for example, through the series of meteorological measurements. However, there are still no statistical models as the dynamics of weather parameters.

At the information technology level, the 23rd Hilbert problem (development of methods of variational calculus) was solved by us [7].

At the same time, **the variation of functions** is reduced to the conscious selection of stable laws and the construction of adequate stable laws on their basis. We adhere to the concept of Descartes on the need to apply an algebraic equation

of General form (1) directly as a finite mathematical solution of unknown differential or integral equations.

RESULTS AND DISCUSSION

After identifying the general model (1) from three coordinates in table 2, six binary relations were obtained.

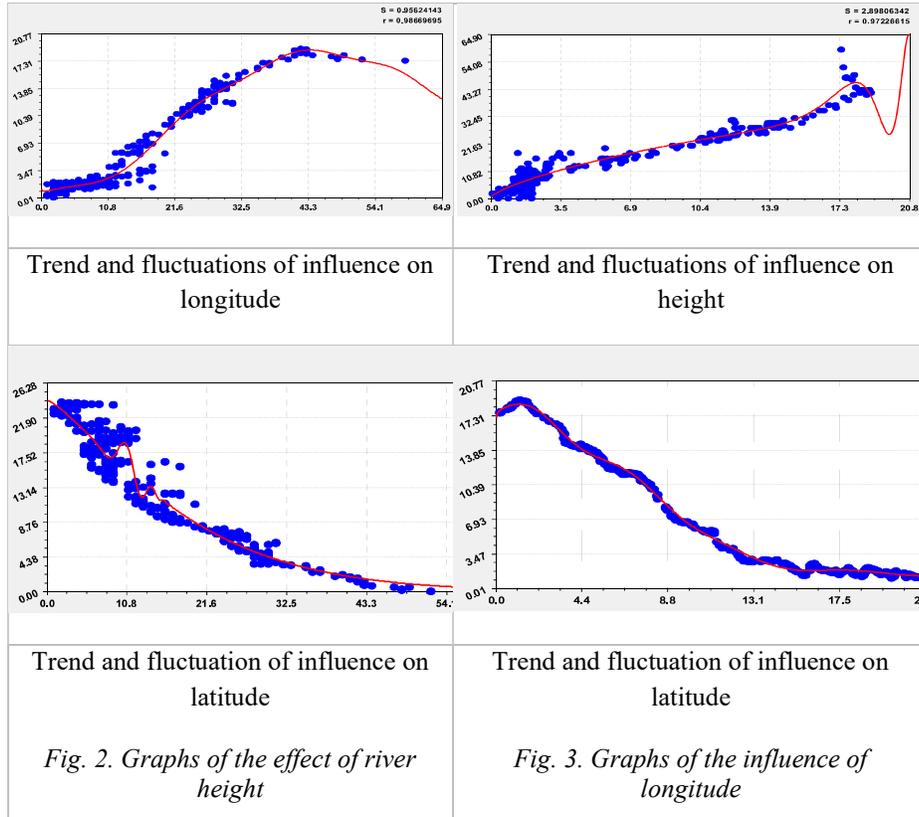
Table 2. Model parameters for binary relations of the coordinates of the Irovka River

| Number <i>i</i> | Wavelet | | | | | | | | Coef. corr. <i>r</i> |
|--|---|----------|------------|----------|------------------|------------|----------|----------|----------------------------|
| | $y_i = a_{1i}x^{a_{2i}} \exp(-a_{3i}x^{a_{4i}}) \cos(\pi x / (a_{5i} + a_{6i}x^{a_{7i}}) - a_{8i})$ | | | | | | | | |
| | Amplitude (half) of oscillations | | | | Half oscillation | | | Shift | |
| | a_{1i} | a_{2i} | a_{3i} | a_{4i} | a_{5i} | a_{6i} | a_{7i} | a_{8i} | |
| The effect of altitude from the mouth to the source of the river on the longitude | | | | | | | | | |
| 1 | 0.036510 | 1.84872 | 0.0001525 | 2.25378 | 0 | 0 | 0 | 0 | 0.9967 |
| 2 | 2.17901 | 0 | 0.060421 | 0.28602 | 10.34394 | -0.0034879 | 1.50771 | 1.02204 | |
| The effect of altitude from the mouth to the source of the river on the latitude North-South | | | | | | | | | |
| 1 | 24.11765 | 0 | 0.021872 | 1.28116 | 0 | 0 | 0 | 0 | 0.9693 |
| 2 | -4.1157e-2 | 33.7806 | 3.19877 | 1 | 10.11218 | -0.14566 | 1.41996 | -3.8745 | |
| The effect of longitude from the left point of the channel on the height from the mouth | | | | | | | | | |
| 1 | 4.58566 | 0.73085 | 0 | 0 | 0 | 0 | 0 | 0 | 0.9723 |
| 2 | 2.03445 | 0 | -1.7537e-5 | 3.90439 | 42.76414 | -1.60613 | 1/03524 | -1.2629 | |
| The effect of longitude from the left point of the channel on the North-South latitude | | | | | | | | | |
| 1 | 22.87329 | 0 | 0.0022749 | 2.42253 | 0 | 0 | 0 | 0 | 0.9724 |
| 2 | -9.48597 | 3.81920 | 2.13995 | 0.64925 | 0 | 0 | 0 | 0 | |
| 3 | 3.10268e8 | 6.37127 | 19.73370 | 0.25275 | 0.41871 | 0.019294 | 1.94415 | 0.14850 | |
| The influence of latitude North-South from source to mouth on the height from the mouth | | | | | | | | | |
| 1 | 58.96563 | 0 | 0.11761 | 0.85812 | 0 | 0 | 0 | 0 | 0.9859 |
| 2 | -1.71136e6 | 0.70048 | 12.14870 | 0.057545 | 0 | 0 | 0 | 0 | |
| 3 | 1.4690e-34 | 38.8795 | 1.67667 | 1.03588 | 83.45225 | -4.18801 | 0.91671 | -2.7359 | |
| The effect of North-South latitude from source to mouth on longitude from the left point | | | | | | | | | |
| 1 | 19.08116 | 0 | 0.039811 | 1.32388 | 0 | 0 | 0 | 0 | 0.9976 |
| 2 | -1.6119e-6 | 26.2539 | 14.89933 | 0.49461 | 0 | 0 | 0 | 0 | |
| 3 | -2.09089 | 0 | 0.82090 | 0.40884 | 2.36913 | 0.0028086 | 1.55183 | -0.3599 | |

The greatest adequacy with a correlation coefficient of 0.9976 was obtained by the influence of latitude on longitude. In second place with correlation 0.9967 was the influence of the height of the points of the channel of the small river on local longitude, and it is mainly determined by the relief of the Vyatka Uval. In third place was the influence of latitude on height with a correlation coefficient of 0.9859.

And in last sixth place is the inverse effect of altitude on local latitude in the North-South direction. All two-three-membered models have the strongest adequacy, that is, the correlation coefficient is more than 0.95.

The influence of altitude from the mouth to the source of the river and longitude from the left point of the channel. Figures 2 and 3 show graphs of the influence of altitude and longitude for the models from table 2.



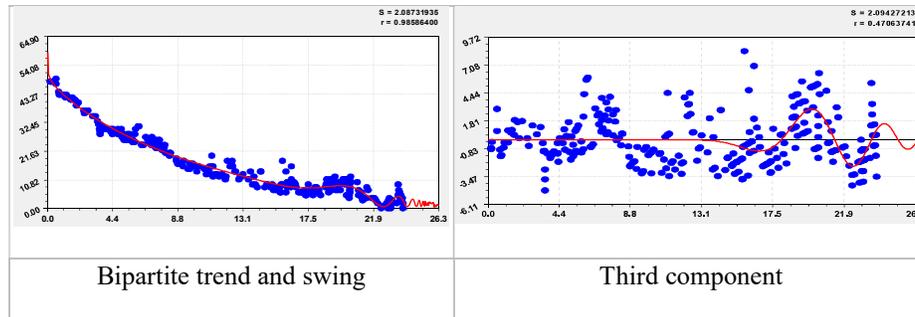
High adequacy allows us to conclude that the coordinates of the small river bed have common properties. One of them is the continuity of the water flow.

The impact of North-South latitude from source to mouth. Latitude affects the other two parameters (altitude and longitude) fractally (Table 3 includes 11 members, Table 4 - 13 wavelets). This fractality is determined by the multiple structure of the components (1).

Table 3. The influence of North-South latitude from source to estuary of the height of the Irovka River

| Number <i>i</i> | Wavelet | | | | | | | | Coef. corr. <i>r</i> |
|--------------------|---|----------|----------|----------|------------------|-----------|----------|----------|----------------------------|
| | $y_i = a_{1i}x^{a_{2i}} \exp(-a_{3i}x^{a_{4i}}) \cos(\pi x / (a_{5i} + a_{6i}x^{a_{7i}}) - a_{8i})$ | | | | Half oscillation | | | Shift | |
| | a_{1i} | a_{2i} | a_{3i} | a_{4i} | a_{5i} | a_{6i} | a_{7i} | a_{8i} | |
| 1 | 58.96563 | 0 | 0.11761 | 0.85812 | 0 | 0 | 0 | 0 | 0.9859 |
| 2 | -1.71136e6 | 0.70048 | 12.14870 | 0.057545 | 0 | 0 | 0 | 0 | |
| 3 | 1.46900e-34 | 38.87949 | 1.67667 | 1.03588 | 83.45225 | -4.18801 | 0.91671 | -2.7359 | |
| 4 | 0.156044 | 3.10878 | 0.486002 | 1 | 2.82186 | 1.2071e-4 | 2.76209 | 1.22940 | 0.3909 |
| 5 | 2.89862e-15 | 20.01880 | 1.28894 | 1.01392 | 0.071542 | 0.0097915 | 1.07050 | 13.4560 | 0.3601 |
| 6 | 5.09823e-99 | 116.0002 | 2.35448 | 1.33230 | 0.077773 | 0 | 0 | 4.71515 | 0.2785 |
| 7 | 5.26586e-6 | 4.85253 | 0.153153 | 0.990526 | 0.106066 | 0 | 0 | 1.39946 | 0.1704 |
| 8 | 2.71862e-81 | 0 | 4.18864 | 1 | 0.113739 | 2.66500 | 0 | 0 | 0.1474 |
| 9 | 5.20858e-23 | 35.73529 | 3.03928 | 1.01566 | 4.04816 | 0.0655608 | 1.23614 | -3.8501 | 0.1035 |
| 10 | 0.0921386 | 1.56575 | 0.172271 | 1 | 0.524478 | 4.53850 | 0 | 0 | 0.1791 |
| 11 | 0.896703 | 2.36906 | 0.877726 | 0.901428 | 0.771798 | 0 | 0 | 2.92625 | 0.1891 |

The adequacy of the influence of latitude can reach a correlation coefficient of 1. And this fact shows that the influence of local latitude has a high degree of certainty of quantization by wave equations (Fig. 4-7). This is called full factor analysis.



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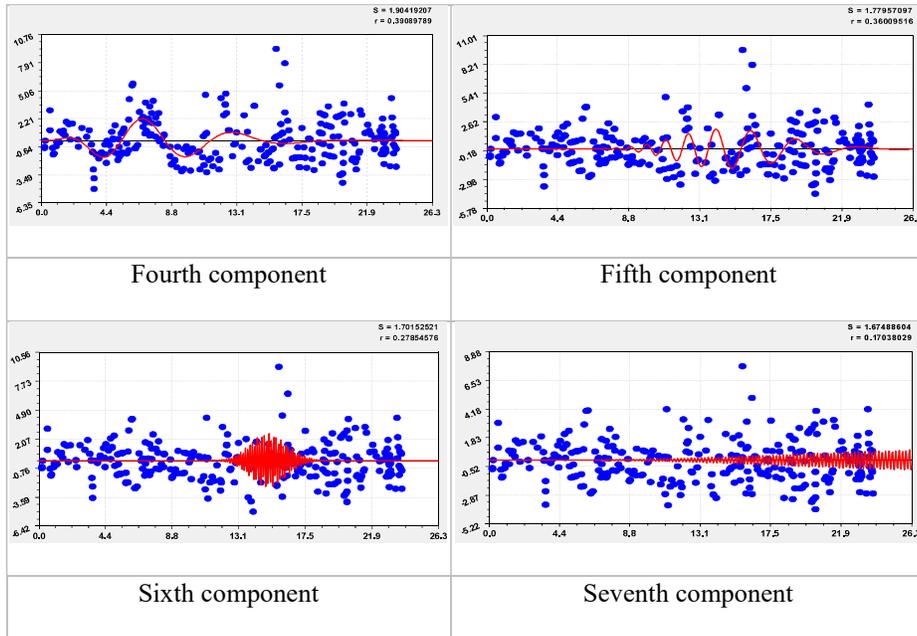


Fig. 4. Graphs of the effect of North-South latitude from source to mouth on the height of the Irovka River

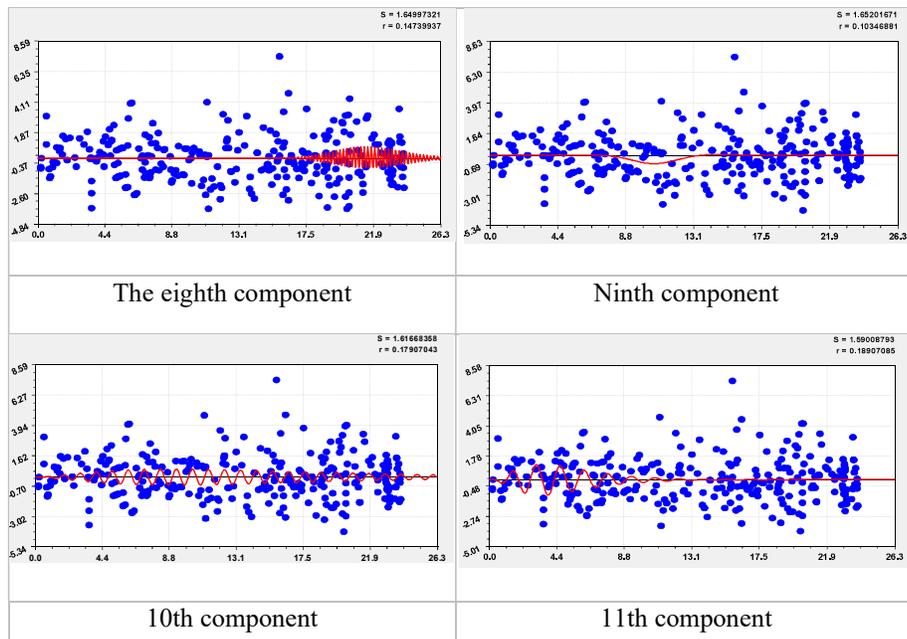
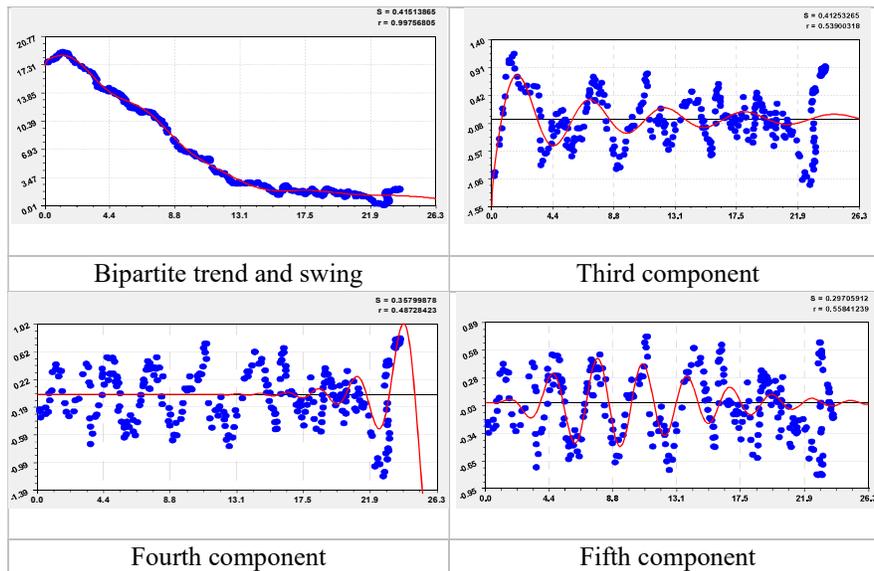


Fig. 5. Graphs of the influence of the North-South latitude on the height from the mouth to the source of the Irovka River

Table 4. The effect of North-South latitude from source to mouth on the longitude of the Irovka River

| Number <i>i</i> | Wavelet | | | | | | | | Coef. corr. <i>r</i> |
|--------------------|---|----------|-----------|----------|------------------|-----------|----------|----------|----------------------------|
| | $y_i = a_{1i}x^{a_{2i}} \exp(-a_{3i}x^{a_{4i}}) \cos(\pi x / (a_{5i} + a_{6i}x^{a_{7i}}) - a_{8i})$ | | | | | | | | |
| | Amplitude (half) of oscillations | | | | Half oscillation | | | Shift | |
| a_{1i} | a_{2i} | a_{3i} | a_{4i} | a_{5i} | a_{6i} | a_{7i} | a_{8i} | | |
| 1 | 19.08116 | 0 | 0.039811 | 1.32388 | 0 | 0 | 0 | 0 | 0.9976 |
| 2 | -1.6119e-6 | 26.2539 | 14.89933 | 0.49461 | 0 | 0 | 0 | 0 | |
| 3 | -2.09089 | 0 | 0.82090 | 0.40884 | 2.36913 | 0.0028086 | 1.55183 | -0.35993 | |
| 4 | 1.95823e-5 | 0 | 0.448432 | 1.00282 | 0.0820466 | 0.0119878 | 1.00090 | 5.45636 | 0.4873 |
| 5 | 0.0168527 | 3.08551 | 0.378919 | 1.00055 | 1.58592 | 6.8921e-5 | 2.25901 | 2.74470 | 0.5584 |
| 6 | 4.51507e-6 | 4.65846 | 0.0012670 | 2.65421 | 0.156256 | 0.232502 | 0.21971 | 12.0842 | 0.3172 |
| 7 | 0.0074286 4 | 0 | 2.07429 | 0.127857 | -0.033421 | 1.99360 | 0.07957 | 6.22674 | 0.3006 |
| 8 | 1.14486e-8 | 7.88750 | 0.253085 | 1.12234 | 5.64354 | 1.01837 | 0 | 0 | 0.2100 |
| 9 | 1.59713e-7 | 5.97718 | 0.379652 | 0.825954 | 0.857850 | 0 | 0 | 6.11295 | 0.2035 |
| 10 | 1.28249 | 0.78853 | 1.19686 | 0.595343 | 0.772734 | 0.0263791 | 0.91301 | 1.77834 | 0.3538 |
| 11 | 1.7925e-13 | 11.7651 | 0.564683 | 0.905328 | 0.375291 | 0 | 0 | 2.47176 | 0.1360 |
| 12 | 0.0264789 | 1.10650 | 0.119784 | 0 | 0.497133 | 0 | 0 | 1.05946 | 0.2198 |
| 13 | 1.31795e-8 | 11.2761 | 2.044278 | 0.735657 | 0.594690 | 0 | 0 | 0.97851 | 0.0465 |



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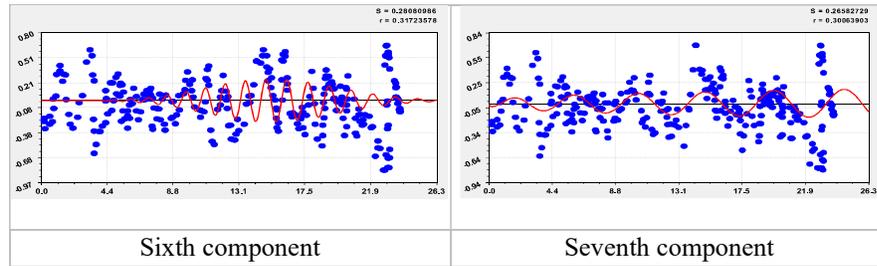


Fig. 6. Graphs of the influence of North-South latitude on longitude from the left point of the channel of the Irovka River

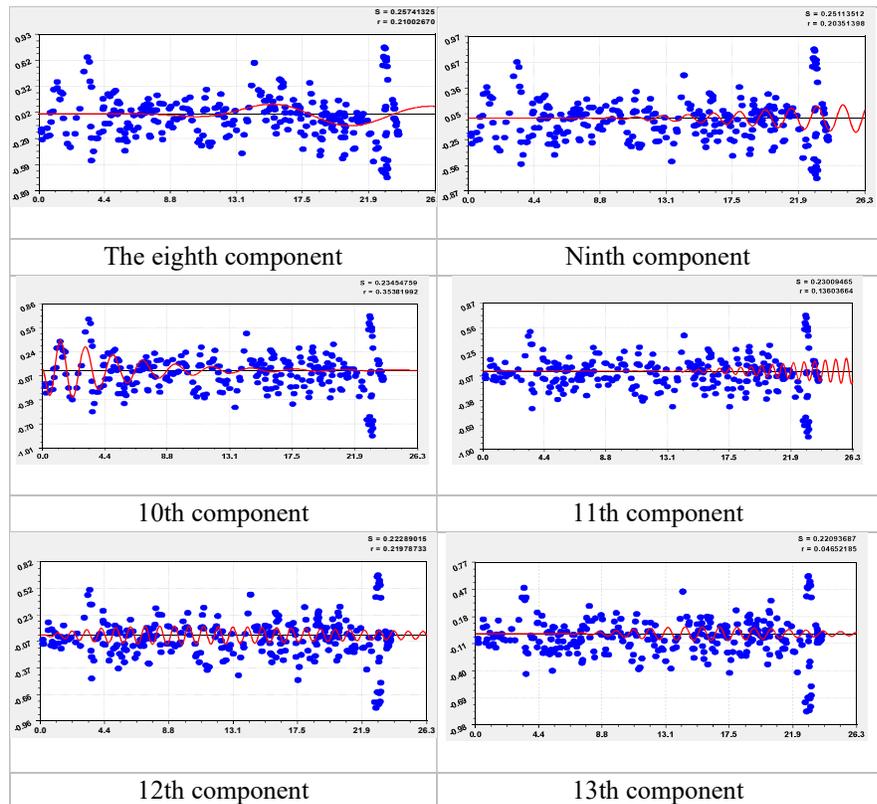


Fig. 7. Graphs of the influence of North-South latitude on longitude from the left point of the channel of the Irovka River

Thus, of the three coordinates for 290 characteristic points of the channel of the small river Irovka, the largest level of non-multiple fractality has the set of 13 wavelets of the influence of the local North-South latitude on the local longitude. A comparison of rank distributions showed that model (1) for latitude received 25 members, for longitude 18 and for a height of 12 members.

CONCLUSION

The hypsometric characteristic is the most important property of the relief. The Irovka River belongs to the low level, at the mouth it is 89 m high, and at the source it is 148 m above sea level. The article shows the possibility of obtaining patterns of hypsometry.

Modeling of binary relations between the three coordinates (latitude, longitude, and height) of the characteristic points of the Irovka river showed that the local latitude receives the greatest quantum certainty. In this case, all paired regularities received a correlation coefficient of more than 0.95. Such a high adequacy of wave patterns shows that geomorphology can go over to multiple fractal representation of the relief. The Irovka River is characterized by a small anthropogenic impact, therefore, the relief over a length of 69 km has the natural character of the oscillatory adaptation of a small river to the surface of the Vyatka Uval from its eastern side. This allows us to proceed to the analysis of the four tributaries of the small river Irovka.

The greatest adequacy with a correlation coefficient of 0.9976 was obtained by the influence of latitude on longitude, that is, the geographical location of the relief of the river channel relative to the geomorphology of the land on the eastern side of the Vyatka Uval. In second place with a correlation of 0.9967 was the influence of the height of the points of the channel of the small river on local longitude, and it is also mainly determined by the relief of the Vyatka Uval. In third place was the influence of latitude on height with a correlation coefficient of 0.9859. And in last sixth place is the inverse effect of altitude on local latitude in the North-South direction.

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