WAVELET ANALYSIS OF HEIGHT AT POINTS OF THE CHANNEL OF A SMALL RIVER FROM THE MOUTH ON SPACE IMAGES

Dr. Sc., Prof. Peter Matveevich Mazurkin [1]
Yana Oltgovna Georgieva [2]
[1, 2] Volga State University of Technology, Yoshkar-Ola, Russia

ABSTRACT

The purpose of the article is to analyze asymmetric wavelets of changing the coordinate - the local elevation of the Irovka river channel, as well as the influence of local latitude and longitude on it of 290 characteristic points along the river channel from source to mouth. After identifying the general wavelet equation, 12 terms were obtained in the form of asymmetric wavelets with variable amplitude and oscillation period. The first three members gave a correlation coefficient of 0.9993, which is slightly less than for latitude 0.9999, but more than for longitude 0.9991. As a result, all three coordinates have the strongest factorial relation with an adequacy of more than 0.999. All 12 wavelets distribute 95.52% of the points of the channel with a model error of up to 5%. The first term of the error distribution formula at 290 points of the river bed is the Laplace (Mandelbrot, Zipf-Perl, Pareto) law of exponential decline, and the second equation shows the stress excitation of the number of errors according to the biotechnical law. The comparison showed that 25 fractal terms were obtained for latitude, 18 for longitude, and 12 wavelets for altitude. As a result, height as a factor also shows high certainty in the quantization of wave equations. A zero rank in the fractal distribution of 12 wavelets receives an arithmetic mean value. The standard deviation decreases from 12.219 for the arithmetic mean to 1.489, that is, 8.2 times, for the first term. After the 12th wavelet, according to the tri-sigma rule, a spread of 0.3 m is formed, which is much less than the actual measurement error of 0.5 m for height from satellite images.

Keywords: *river, satellite image, height, wavelets*

INTRODUCTION

Images transmitted by Earth remote sensing satellites (ERS) are used in many sectors - agriculture, geological and hydrological research, forestry, environmental protection, territorial planning, educational, reconnaissance and other purposes. Remote sensing space systems allow for a short time to obtain the necessary data from large areas (including hard-to-reach and hazardous areas).[1].

To date, there is no single reasonable choice of the type of function. With the development of the slope, the washing and deposition rate of the material at different points of the profile should change. This determines the self-development of slope systems due to feedback, when the slope-dependent diffusion coefficient is responsible for the profile shape itself [2].

Mathematical and geomorphological modeling of erosion landscapes using a geoinformation medium and high-resolution satellite imagery makes it possible to identify the main characteristics of landscapes in real time and use them as the basis for theoretical and experimental studies of ecological and geomorphological processes. Thus, we get a modern tool for studying the processes that determine the state and degree of development of landscapes [3].

The longitudinal profile of the channel flow always has a series of steps or, rather, gentle undulating bends that do not cause a continuity gap like rapids and waterfalls. Graduation as an indispensable feature of channel flows was noted by most researchers [4]. But wave equations have not yet been obtained.

The goal is the analysis of asymmetric wavelets of height distributions from source to mouth at 290 characteristic points and the influence of the latitude and longitude of the small river Irovka.

MATERIALS AND METHODS

From satellite images, coordinates (latitude, longitude, altitude) were measured according to the recommendations [5].

On the rod line of a small river, characteristic points are selected from the source to the mouth according to sharp changes in the longitudinal profile of the small river, for example, when the channel turns in any direction in terms of more than 10-15^0. Based on the measurement results, a data table is compiled for modeling by identifying stable patterns [6]. Table 1 shows the coordinates measurements and their comparison with the models: ε_{12} - residues after the 12th component (1); Δ - relative error.

Table 1. Rank distributions of the height of the characteristic points of the channel of the Irovka River

Point rank	Rank R_h	Latitude α, minute	Longitude β, minute	Height h, м	Estimated values from rank		
					h, м	ε_{12}, м	Δ, %
0	0	0	17.39	59	59.1	0.123212	0.21
1	1	0.02	17.50	52	52.0	-0.0367244	-0.07
2	3	0.19	17.62	48	48.0	0.0170352	0.04
3	3	0.2	17.67	48	48.0	0.0170352	0.04
4	3	0.27	17.64	48	48.0	0.0170352	0.04
…	…	…	…	…	…	…	…
286	247	23.83	1.970	4	4.0	-0.00580508	-0.15
287	247	23.84	2.019	4	4.0	-0.00580508	-0.15
288	269	23.87	2.035	2	2.1	0.0747201	3.74
289	286	23.89	2.017	0	-0.4	-0.422514	$-\infty$

The hypsometric characteristic is one of the most important properties of the relief. By the degree of elevation of the land surface above sea level, a low-lying (absolute height from 0 to 200 m) relief is distinguished [7]. The Irovka River belongs to the low level, at the mouth it is 89 m high, and at the source the height reaches 148 m above the Baltic Sea [8], [9].

RESULTS AND DISCUSSION

Figure 1 shows a spatial hypsometric curve along 290 characteristic points of change in local height from the mouth to the source of the river, depending on local latitude and longitude.

The model is obtained on the basis of the condition that at first one-factor models are identified, and then they are placed to increase the correlation coefficient as a measure of the adequacy of the revealed patterns.

Oscillations (wavelet signals) are written by the wave formula [4] of the form

$$y_i = A_i \cos(\pi x / p_i - a_{8i}),\quad A_i = a_{1i} x^{a_{2i}} \exp(-a_{3i} x^{a_{4i}}),\quad p_i = a_{5i} + a_{6i} x^{a_{7i}}, \qquad (1)$$

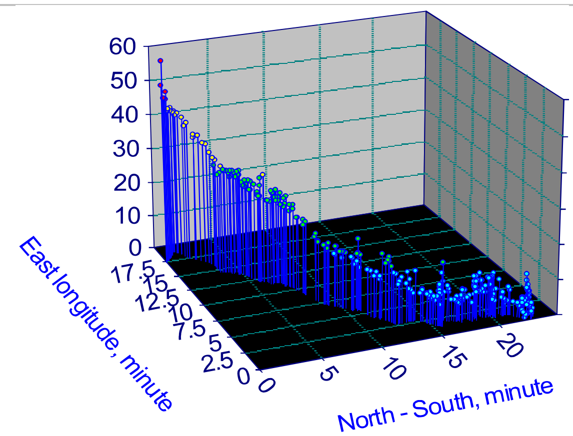


Fig. 1. Spatial hypsometry of the Irovka River by the influence of local latitude and longitude on altitude

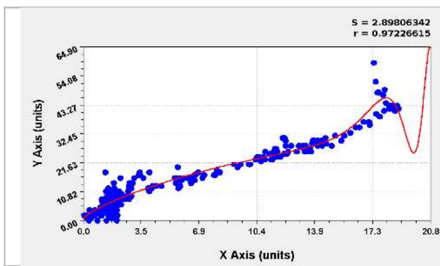
where y - is the indicator (dependent factor), i - is the number of the component of the model (1), m - is the number of terms in the general model (1), x - is the explanatory variable (influencing factor), a_1 a_8 - are the parameters of the model (1) that take different numerical values in the process of structural-parametric identification in the software environment CurveExpert-1.40, A_1 - the amplitude (half) of the wavelet (axis y), p_i - the half-cycle of the oscillation (axis x).

Table 2 shows the values of parameters (1) for two terms of the influence of local longitude on altitude, and the residuals of the second term as an influencing variable are the values of local latitude from the data in Table 1. Figure 2 shows graphs of two factors.

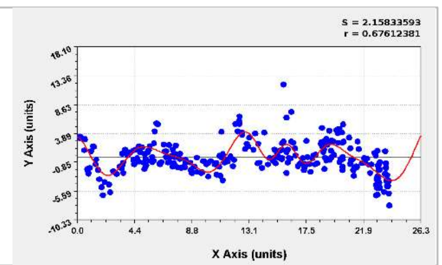
Table 2. The effect of longitude and latitude from source to mouth on the height

Factor and number i	Wavelet $y_i = a_{1i}x^{a_{2i}}\exp(-a_{3i}x^{a_{4i}})\cos(\pi x/(a_{5i}+a_{6i}x^{a_{7i}})-a_{8i})$								Coef. corr. r
	Amplitude (half) of the oscillation				Half oscillation			Shift	
	a_{1i}	a_{2i}	a_{3i}	a_{4i}	a_{5i}	a_{6i}	a_{7i}	a_{8i}	
β_1	4.58566	0.73085	0	0	0	0	0	0	0.9723
β_2	2.03445	0	-1.7537e-5	3.90439	42.7641	1.60613	1.03524	-1.26285	
α_1	2.30115	0	-0.026287	1	2.86126	0.019765	1	0.007428	0.6761
α_2	5.5514e-13	165.938	10.02393	1	0.03908	0.019846	1	2.21030	
α_3	1.54657	0	0	0	2.28105	0.001715	1	-0.77225	

According to the hypsometric picture in Figure 1, the small Irovka River flows in a rectangle 23.89 minutes long (local latitude North-South) and 18.89 minutes wide (local longitude). Sharp changes in the channel curvature in terms of 290 characteristic points gave wave equations according to table 2.



The impact of local longitude



Additionally, the influence of local latitude

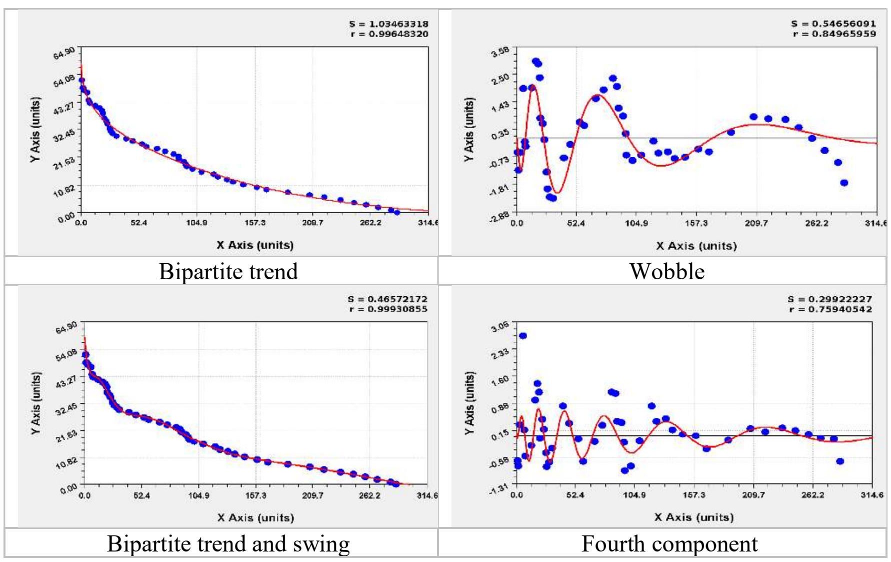
Fig. 2. The influence of longitude and latitude from source to mouth on the height of the channel of the Irovka River

The first equation for the influence of longitude shows that as east longitude increases, the height of the river increases according to the exponential law. It can be seen from the picture in Figure 1 that the increase in altitude occurs from a shorter longitude (closer to the mouth) to a greater longitude (closer to the source). In this case, the second wave equation shows that the amplitude increases according to the exponential law of growth, and the half-period of the oscillation increases from 42.76 ranks at minimum longitude.

Additionally, the influence of latitude on height by the third term occurs according to the law of exponential growth from source to mouth. Therefore, a decrease in altitude occurs mainly due to a decrease in local longitude. This third term from Table 2 is the law of Laplace (in mathematics), Mandelbrot (in physics), Zipf-Perl (in biology) and Pareto (in econometrics). The law a_{4i} = 1. After identifying the general model (1), 12 wavelets of rank distribution of local height were obtained (Table 3, Figs. 3-5).

Table 3. Parameters of wavelets of rank distribution of the local height of the Irovka

Number i	Вейвлет $y_i = a_{1i}x^{a_{2i}} \exp(-a_{3i}x^{a_{4i}})\cos(\pi x/(a_{5i} + a_{6i}x^{a_{7i}}) - a_{8i})$								Coef. corr. r
	Amplitude (half) of the oscillation				Half oscillation			Shift	
	a_{1i}	a_{2i}	a_{3i}	a_{4i}	a_{5i}	a_{6i}	a_{7i}	a_{8i}	
1	58.87679	0	0.0008808	1.09926	0	0	0	0	0.9993
2	-6.38434	0.48652	0.070117	0.46505	0	0	0	0	
3	3.47327	0.84367	1.36494	0.26690	4.78474	0.51777	0.78599	5.06935	
4	0.458135	0.692074	0.459628	0.418576	5.06960	0.168685	0.838476	8.53040	0.7594
5	9.3361e-7	4.10453	0.0650157	1	3.73444	0.0408429	0.489883	0.194488	0.3095
6	0.394701	1.23562	0.447487	0.682474	3.12365	0.551263	0	0	0.4576
7	0.781908	1.36008	0.256209	1.03850	3.16303	0.0119643	2.26069	2.1713	0.4440
8	3.1833e-11	6.49697	0.0664222	1	10.78055	0.0254631	0.996656	1.18507	0.6270
9	6726.89934	2.21488	14.79868	0.099782	4.67037	0.0082493	1.19103	-1.11802	0.4141
10	5.0896e-9	4.247939	0.0018925	1.53230	1.47780	0	0	0.257972	0.3900
11	5.2799e-12	21.44634	2.22290	1	0.832376	0	0	3.50578	0.2969
12	3.2771e-47	2941689	0.214460	1.07043	1.05525	0	0	2.34453	0.5136



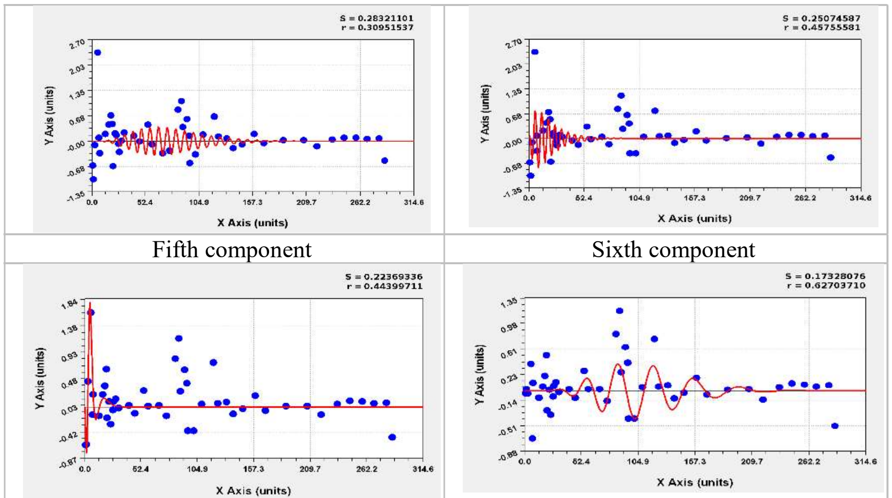
Bipartite trend | Wobble

Bipartite trend and swing | Fourth component

Fig. 3. Graphs of the trend and two fluctuations in the rank distribution of the height

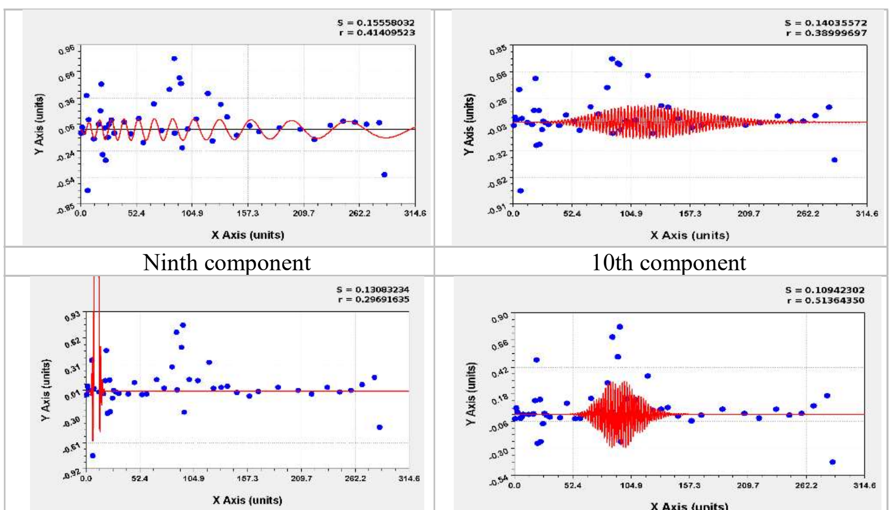
The first three wavelets gave a correlation coefficient of 0.9993 for the capabilities of the CurveExpert-1.40 software environment. The first term is the Laplace or Mandelbrot law [4] modified by us under condition 1, and it shows an exponential decrease in the average height of two banks from source to mouth. The second term shows a decrease in height according to the biotechnical law [4] approximately to the middle of the length of the river. Approximately the first two terms form a trend in the change in the line of the water surface of a small river

(Fig. 3). The remaining 10 waves (Fig. 4 and Fig. 5) show, due to the positive sign, the oscillatory adaptation of the relief to an increase in height. The first two vibrations show the relief from the source.



Fifth component	Sixth component
Seventh component	The eighth component

Fig. 4. Graphs of models of the rank distribution of the height of the Irovka River



Ninth component	10th component
11th component	12th component

Fig. 5. Graphs of the latest models of the rank distribution of the height of the Irovka

The remaining eight members show a local change in the height of the river topography. Thus, the height along the small river changes, like latitude and longitude, in a wave-like fashion, which, of course, is affected by fluctuating distances in plan and in height between the characteristic points along the rod of the small river Irovka.

Figures 4 and 5 show several short wavelets at characteristic points. In addition, members 6, 7, and 11 show strong relief fluctuations in the upper reaches of a small river. With a decrease in the water content at the source in summer, the canal is now drying up.

Further identification of model (1) is difficult, so we stop the process of identifying patterns. The residuals are less than the measurement error of $\pm$0.5 m (Fig. 6).

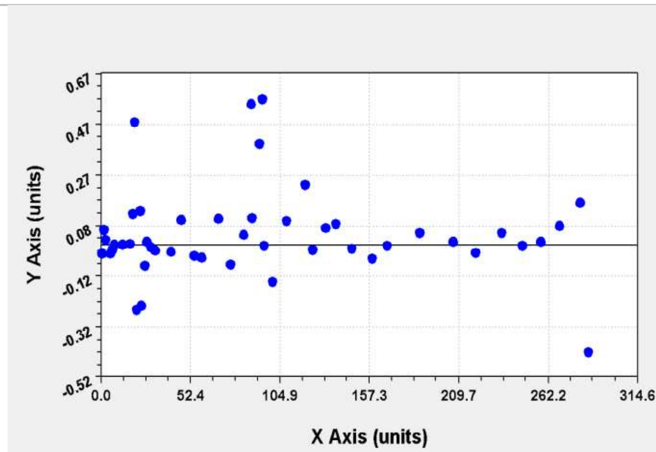


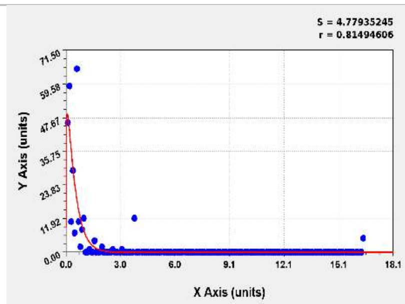
Fig. 6. Remains after the 12th wavelet

Four points (with ranks 252, 263, 266 and 289) due $h = 0$ to gave an infinite error.

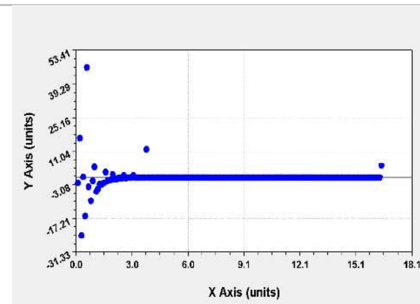
At 0.1 intervals, the error was distributed as follows (modulo): at zero, 9 pcs. (3.10%); 0 - 0.1% 46 pcs. (15.86%); 0.1 - 0.2% 59 pcs. (20.34%); 0.2 - 0.3% 11 pcs. (3.79%); 0.3 - 0.4% 29 pcs. (10%); 0.4 - 0.5% 7 pcs. (2.41%) etc. At large intervals: 0 - 1.0% of all points 203 pcs. (70%); from 0 to 5% 277 points, which is equal to 95.52% of 290 characteristic points. We accept the norm of measurement and modeling errors of 5%, then the remaining 290 - 277 = 13 points of the longitudinal channel profile require separate consideration.

Figure 7 shows a graph of the distribution of the model error by the formula

$$n = 9.00800\exp(-2.47276\Delta_{0.1}) + 84.75096\Delta_{0.1}^{0.20200}\exp(-2.54418\Delta_{0.1}) \quad (2)$$



Bi-term error trend



Residuals after the trend

Fig. 7. Graphs of the distribution of the error in modeling the height of the Irovka River

The first term (2) is the Laplace law of exponential decay, and the second equation shows the stress excitation [4] of the number of errors. The comparison showed that model (1) for latitude received 25 members, for longitude 18 and for a height of 12 members.

Table 4 gives the wavelet ranks and the calculated values of the actual distribution of the standard deviation.

Table 4. Dynamics S, m

Wavelet rank i	Standard deviation S
0	12.219
1	1.489
2	1.035
3	0.466
4	0.299
5	0.283
6	0.251
7	0.224
8	0.173
9	0.156
10	0.140
11	0.131
12	0.109

In the upper right corner of the graphs, the figures show the mean square deviation (standard deviation) S.

Zero rank gets the equation $y=a$ of arithmetic mean value. The first rank has the first member in table 3.

The standard deviation decreases from 12.219 for the arithmetic mean equation to 1.489, that is, 8.2 times. After the 12th wavelet, a three-fold deviation forms 0.3 m, which is much less than the actual measurement error of $\pm$0.5 m.

After identifying the model (1), the formula (Fig. 8) is obtained

$$S = 12.21824\exp(-2.51054i) +$$

$$+1.68900i^{2.35427}\exp(-1.18462i). \qquad (3)$$

The first term of equation (3) is again the law of Laplace (in mathematics), Mandelbrot (in physics), Zipf-Perl (in biology) Pareto (in econometrics) and therefore shows the multiple fractal distribution of all 12 wavelets. Then the Mandelbrot fractals become special cases.

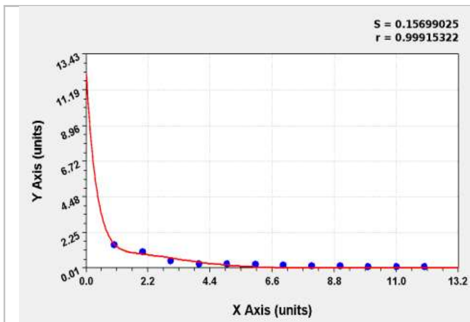
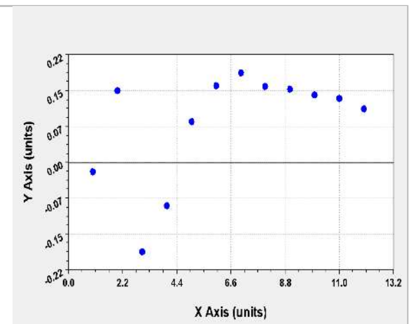
	
Bi-term standard deviation trend	Balance after the trend (3)

Fig. 8. Distribution graphs of the standard deviation of height from the wavelet rank

It can be seen from the residuals in Fig. 8 that the asymmetric wavelet will become the third term in formula (3). This proves that quantization of the relief height by fractals can have not only multiple values, but even occurs according to wave equations.

CONCLUSION

Modeling the distributions of the height reduced to a small river made it possible to obtain 12 wavelets with a distribution of 95.52% of the points with a modeling error of up to 5%. The first term in the formula for the distribution of errors at 290 points of the river bed is the Laplace law of exponential decline, and the second equation shows the stress excitation [4] of the number of errors. The comparison showed that model (1) for latitude received 25 members, for longitude 18 and for a height of 12 members.

The first three members of the height model gave a correlation coefficient of 0.9993, an adequacy level of more than 0.9 or the strongest link. The remaining 9 members increase the level of adequacy to almost 1. As a result, height as a factor shows certainty in quantization.

The zero rank in the fractal distribution receives the equation $y = a$ of the arithmetic mean value. The standard deviation decreases from 12.219 for the arithmetic mean equation to 1.489, that is, 8.2 times. After the 12th wavelet, a three-fold deviation (the tri-sigma rule) forms 0.3 m, which is much less than the actual measurement error of 0.5 m for the height from satellite images.

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